

Equations, linear equations and systems of equations

1. Identify which of these equations is linear and which is non-linear.

- a. $3.235x + 4.57y = 4.7z$
- b. $3.235x^2 + 4.57y = 4.7z$
- c. $3.235x + 4.57xy = 4.7z$
- d. $3.235x + 4.57y = 4.7 \sin(z)$
- e. $e^x + \sin(x) = 5$

2. Suppose you had the equation $x/z + y/z = 1$, could you change this into a linear equation? Similarly, suppose you had $1/x + 1/y = 2$, could you change this into a linear equation?

3. Suppose you had a linear equation $3.52x + 4.57y - 2.51\sin(z) = 4.71$. If the $\sin(z)$ comes from the fact that there is some sort of swinging motion (say, a pendulum), and you know that z will be small, so could you use the approximation $\sin(z) \approx z$ to transform this into a linear equation?

4. Suppose the transformed linear equation in Question 3 was included in a system of three linear equations that had a unique solution. Now, suppose that your solution was:

$$x = 1.352, y = 71.324 \text{ and } z = 0.243$$

Would you say this is an acceptable solution?

5. Suppose, instead, the transformed linear equation in Question 3 was included in a system of three linear equations that had a unique solution. Now, suppose that your solution was:

$$x = 1.352, y = 71.324 \text{ and } z = 2.543$$

Would you say this is an acceptable solution to the system of equations if we did not replace $\sin(z)$ with z ?

6. To ten decimal digits of precision, a solution (there are many due to the periodic nature of sine) to

$$\begin{aligned} 12.1\sin(x) + 4.7\sin(y) &= 0.3 \\ 1.5\sin(x) - 2.3\sin(y) &= 0.5 \end{aligned}$$

is $x = 0.08726668400$ and $y = -0.1612483210$. The unique solution to the system of two linear equations

$$\begin{aligned} 12.1x + 4.7y &= 0.3 \\ 1.5x - 2.3y &= 0.5 \end{aligned}$$

is $x = 0.08715596330$ and $y = -0.1605504587$. Do you think this approximation of $\sin(x) = x$ and $\sin(y) = y$ is acceptable?

What is the relative error of the approximation (the second solution) when compared to the exact solution? Use $\|\mathbf{u} - \mathbf{v}\|_2 / \|\mathbf{u}\|_2$ where $\mathbf{u}$ is the exact solution and $\mathbf{v}$ is the approximation.